ARCOS Group

uc3m | Universidad Carlos III de Madrid

Lesson 2 (I)
Representation of information

Computer Structure

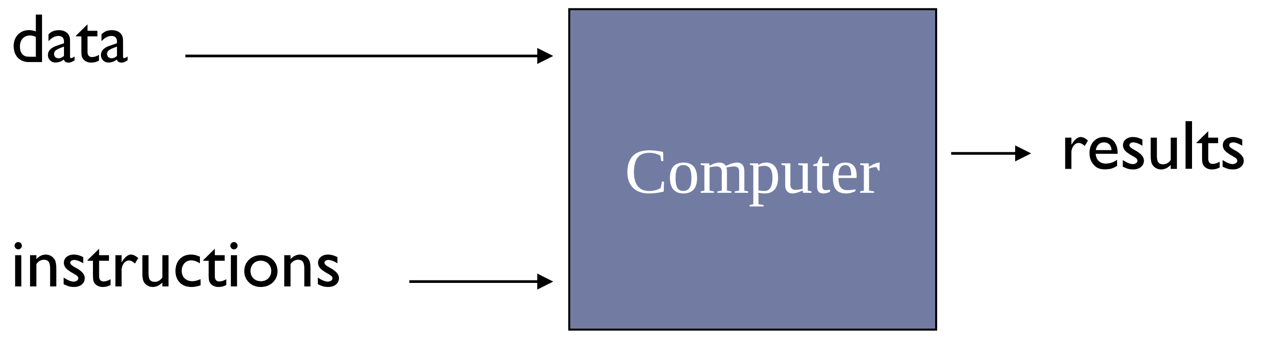
Bachelor in Computer Science and Engineering

Contents

Contents

Introduction
Computer

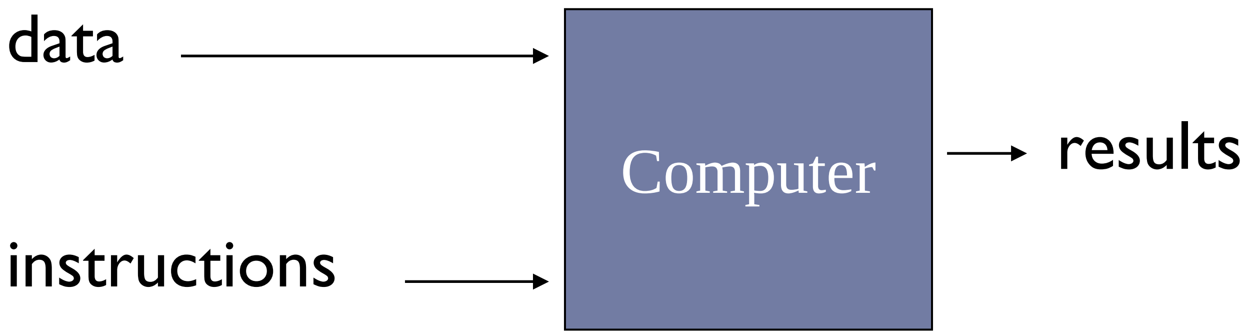
- A computer is a machine designed to process data.



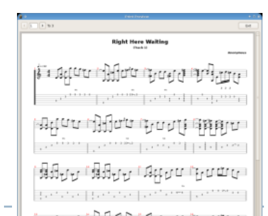
- Instructions are applied and results are obtained.

Introduction
Computer

- A computer is a machine designed to process data.

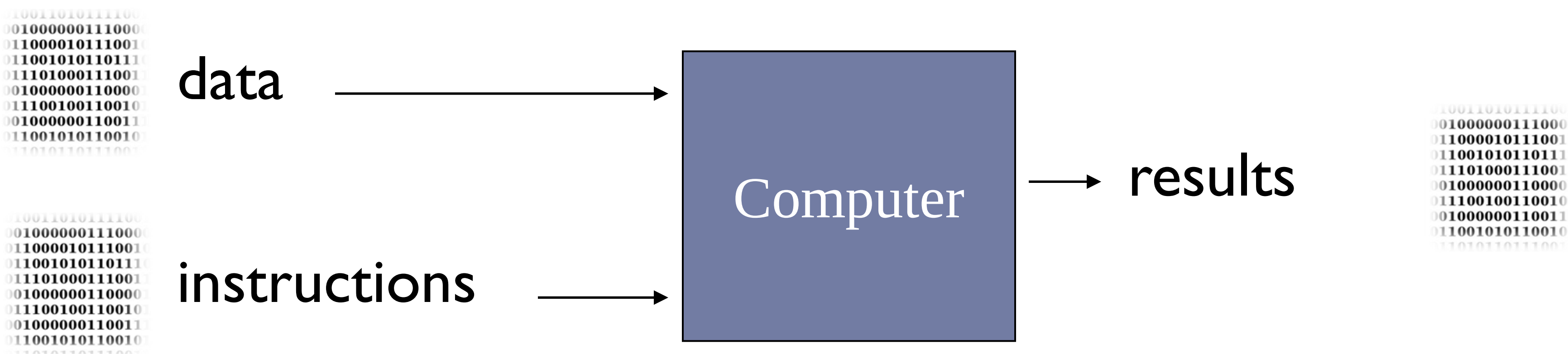


- Instructions are applied and results are obtained.
- The data/information can be of different types.



Introduction
Computer

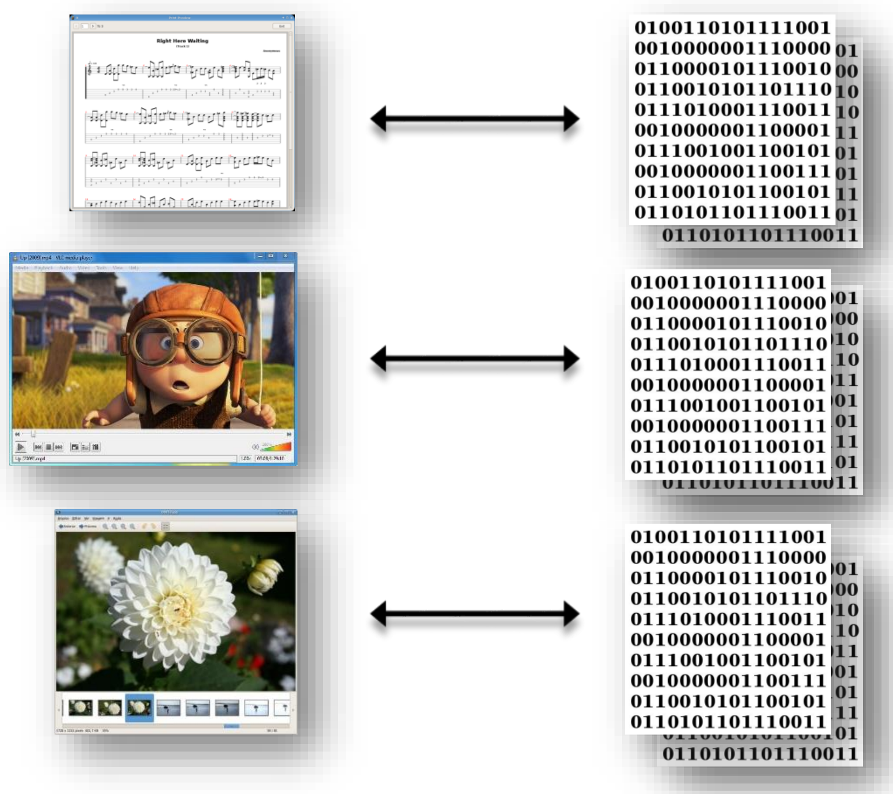
- A computer is a machine designed to process data.



- Instructions are applied and results are obtained.
- The data/information can be of different types.
- A computer uses only one representation: binary.

Introduction
Information representation

- The use of a representation allows the transformation of different types of information into binary (and vice versa).



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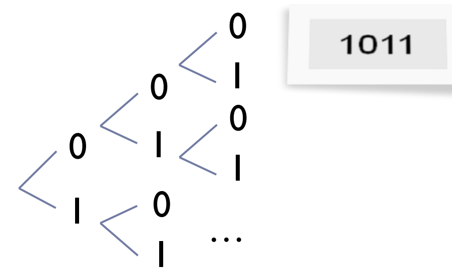
Introduction
Characteristics of the information representation

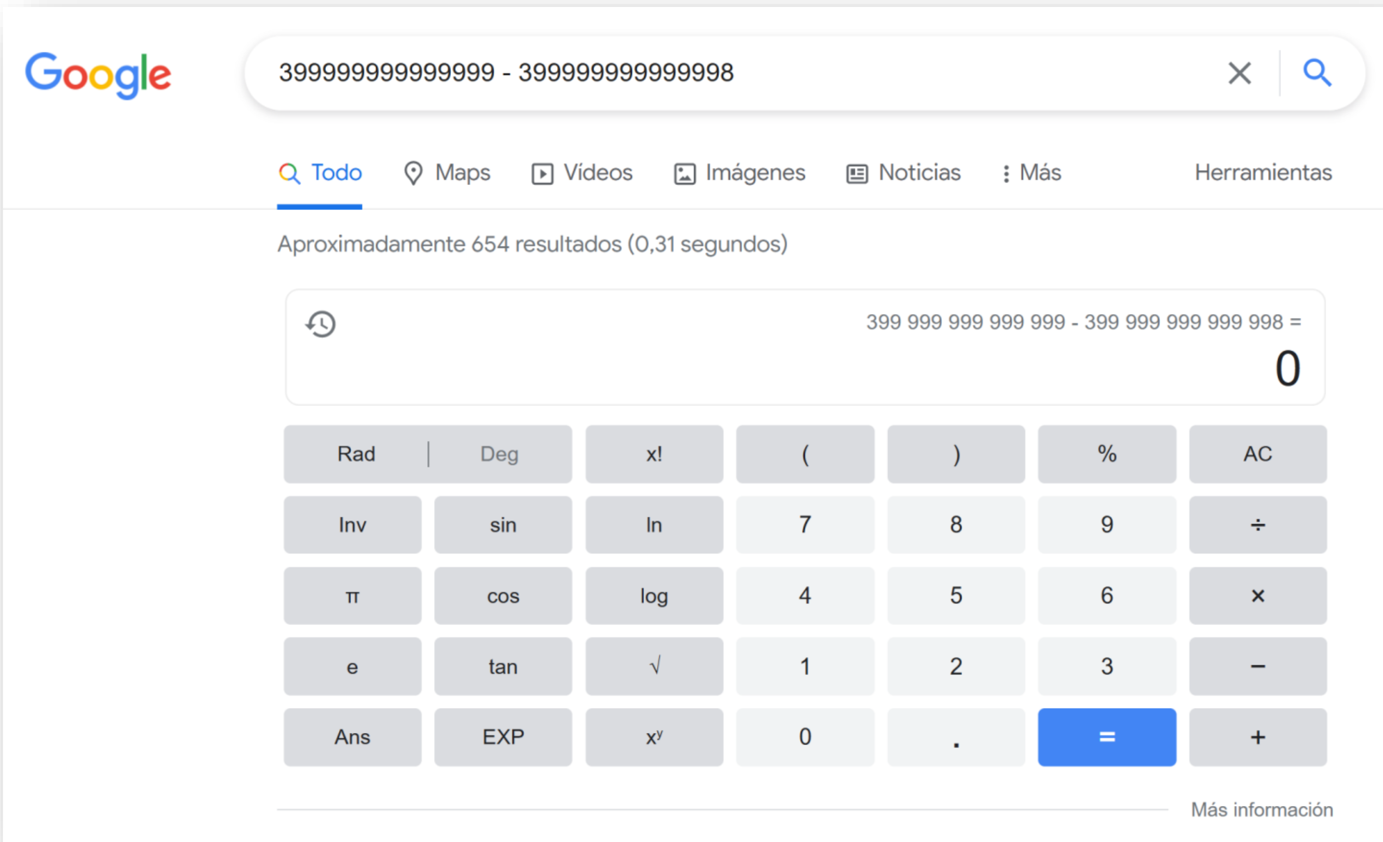
- A computer handles a finite set of values
  - Binary type (two states)
  - Finite (bounded representation)
    - Number of bits of the computer word (32/64)
      or bit (1), nibble (4), byte (8), half w., double w., …
    - With **n** bits, 2^n different values can be encoded



- There are some types of information that are infinite
  - Impossible to represent
    all values of natural numbers,
    real numbers, etc.

- The chosen representation has limitations.

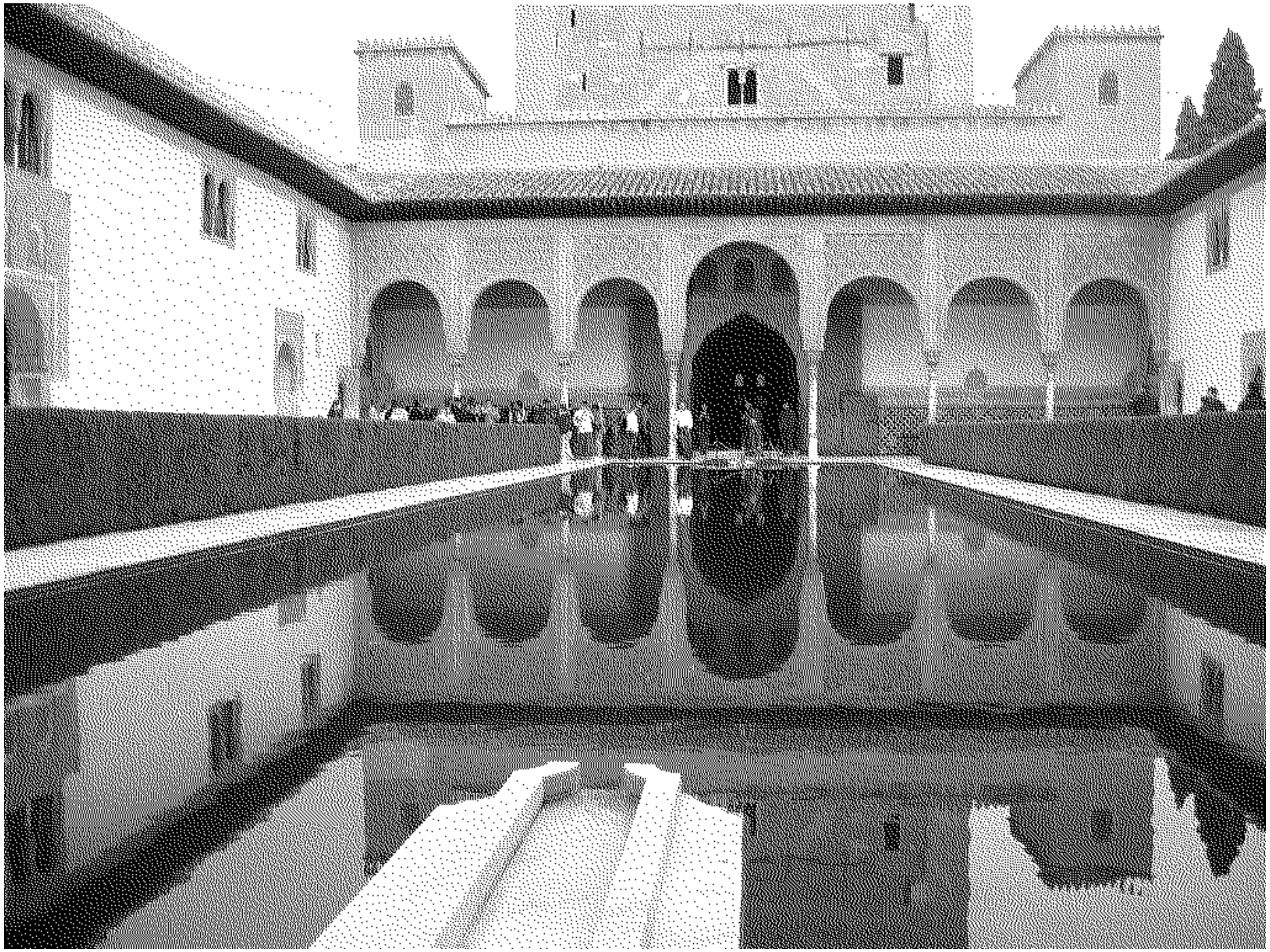
Example 1:
the Google calculator with 15 digits…



http://www.20minutos.es/noticia/415383/0/google/restar/error/

Example 2:
color depth…

1 bit	2 colors
4 bits	16 colors
8 bits	256 colors



http://platea.pntic.mec.es/~lgonzale/tic/imagen/conceptos.html

Example 2:
color depth…

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color depth…

1 bit	2 colors
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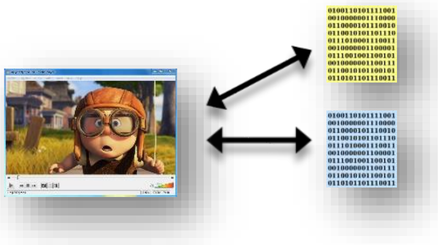
We need…

- To know possible representations:



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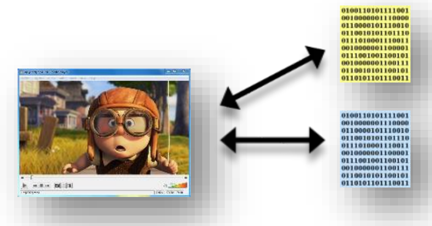
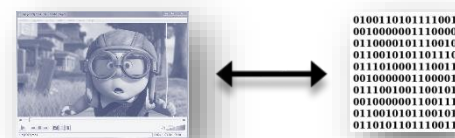


- To know the characteristics of theses representations:
  - Limitations



We need…

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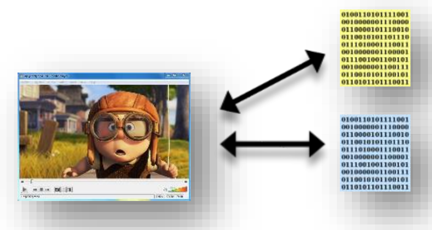
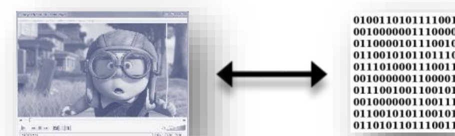
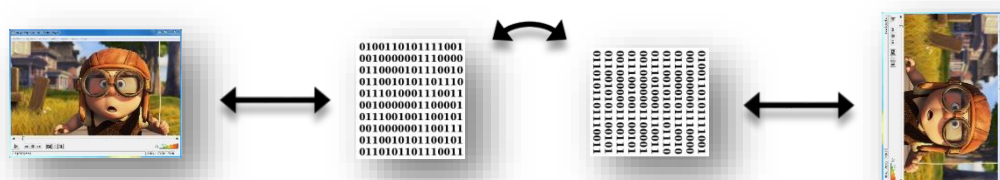


- To know the characteristics of theses representations:
  - Limitations



- To know how work with the selected representation:



Example of failure…

- **Ariane 5** explosion (first flight)
  - Sent by ESA in June 1996
  - Cost of development:
    10 years and 7 billion dollars
  - Exploded 40 seconds after launch, at 3700 meters altitude.
  - Failure due to total loss of altitude information:
    - The inertial reference system software performed the conversion of a 64-bit floating point real value to a 16-bit integer value.
    - The number to be stored was greater than 32767 (the largest 16-bit signed integer) and a conversion failure and exception occurred.

Contents

Positional representation systems

- A number is defined by a ordered list of digits, each of which is affected by a scaling factor that depends on the position it occupies in the list.

- Given a numbering base b,
  a number X is defined as the list of digits:
  $X = (\ldots\ x_2\ x_1\ x_0\ ,\ x_{-1}\ x_{-2}\ \ldots)_b$ Con $0 \leq x_i < b$
  with a list of associated weights:
  $P = (\ldots\ b^2\ b^1\ b^0\ \ b^{-1}\ b^{-2}\ \ldots)_b$

Positional representation systems

- A number is defined by a ordered list of digits, each of which is affected by a scaling factor that depends on the position it occupies in the list.

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  with a list of associated weights:
  $P = (\dots \; b^2 \; b^1 \; b^0 \quad b^{-1} \; b^{-2} \; \dots)_b$

- It value is:

$$V(X) = \sum_{i=-\infty}^{+\infty} b^i \cdot x_i = \cdots b^2 \cdot x_2 + b^1 \cdot x_1 + b^0 \cdot x_0 + b^{-1} \cdot x_{-1} + b^{-2} \cdot x_{-2} \cdots$$

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Positional representation systems

- Decimal

  X = 9 7 3 1

  ... 10^3 10^2 10^1 10^0

- Binary

  X = 0 1 0 1

  ... 2^3 2^2 2^1 2^0

- Hexadecimal

  X = 1 F A 8

  ... 16^3 16^2 16^1 16^0

Positional representation systems

- Decimal

  X = 9 7 3 1

  … 10^3 10^2 10^1 10^0

- Binary

  X = 0 1 0 1

  … 2^3 2^2 2^1 2^0

- Hexadecimal

  X = 1 F A 8

  … 16^3 16^2 16^1 16^0

From binary to hexadecimal:

- Group by 4 bits, right to left
- Each 4 bits is the value of a hexadecimal digit
- E.g.: 1 0 1 0 0 1 0 1

  0x A 5

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Positional representation systems

- Decimal

  X = 9 7 3 1

  … 10^3 10^2 10^1 10^0

¿?

- Binary

  X = 0 1 0 1

  … 2^3 2^2 2^1 2^0

- Hexadecimal

  X = 1 F A 8

  … 16^3 16^2 16^1 16^0

Exercise

1 minute máx.

- To represent 342 in binary:

256	128	64	32	16	8	4	2	1
?	?	?	?	?	?	?	?	?

Exercise (solution)

- To represent 342 in binary:

256	128	64	32	16	8	4	2	1
1	0	1	0	1	0	1	1	0

342-256=86 86-64=22 22-16=6 6-4=2 2-2=0

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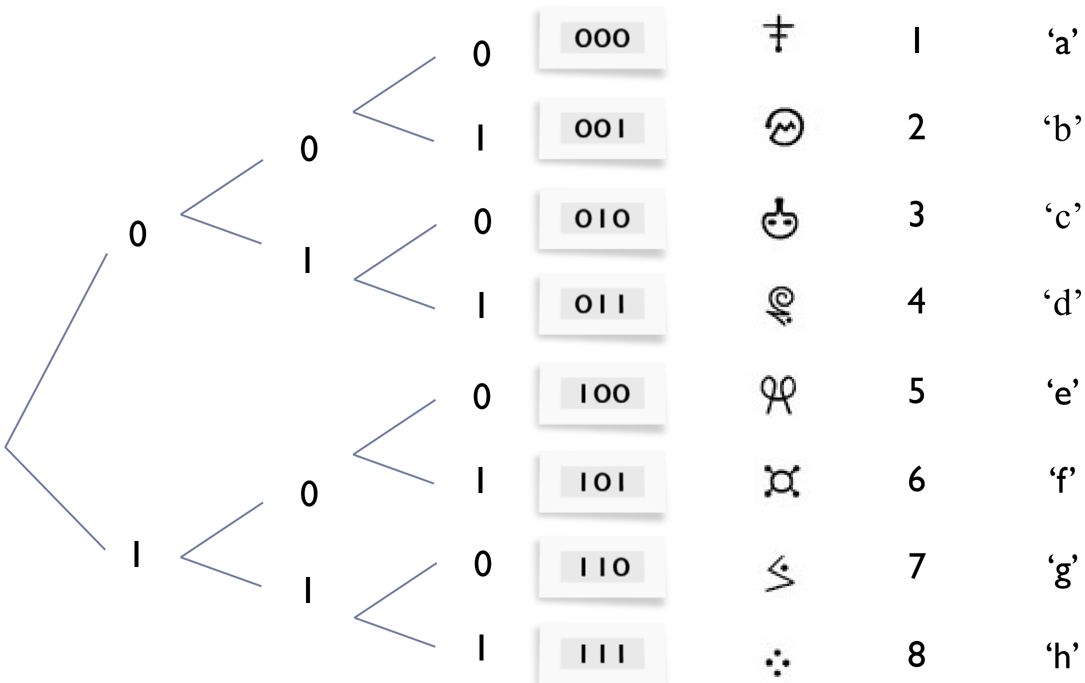
Positional representation systems

- With 3 binary digits, up to 8 symbols can be represented:



Positional representation systems

- How many values can be represented with n bits?

- How many bits are needed to represent m ’values’?

- With n bits, if the minimum representable value corresponds to the number 0,
  what is the maximum representable numerical value?

Positional representation systems

- How many values can be represented with n bits?
  - **2^n**

  - E.g.: with 4 bits up to 16 values can be represented

- How many bits are needed to represent m 'values'?
  - **$\lceil \text{Log2(n)} \rceil$** (Log2(n) round up)
  - E.g.: 6 bits are required to represent 35 values

- With n bits, if the minimum representable value corresponds to the number 0,
  what is the maximum representable numerical value?
  - **2^n-1**

Exercise

10 seconds máx.

- To compute the value of (23 ones):

$$11111111111111111111111_2$$

Exercise (solution)

- To compute the value of (23 ones):

$$11111111111111111111111_2$$

$$X = 2^{23} - 1$$

Tip:

$$\begin{array}{rl} 11111111111111111111_2 & = X \\ +\ 00000000000000000001_2 & = 1 \\ \hline 100000000000000000000_2 & = 2^{23} \end{array}$$

$$X = 2^{23} - 1$$

Example: operations

- Add in binary:

	1	1	1			
		1	0	1	0	0
+		1	1	1	1	0
	1	1	0	0	1	0

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Example: operations

- **Add** in binary:

1	1	1			
	1	0	1	0	0
+	1	1	1	1	0
1	1	0	0	1	0

- **Subtract** in binary:

				1 →	1 →
	0	1	1	0	0
-	0	1	0	1	1
	0	0	0	0	1

Exercise

2 minutes máx.

You have a 5 liter bottle and a 3 liter bottle.
How can you get 4 liters just right?

Exercise (solution)

2 minutes máx.

You have a 5 liter bottle and a 3 liter bottle.
How can you get 4 liters just right?

- Fill the 5-liter jar
- Empty it into the 3-liter jar
  - There are 2 left in the 5-liter jar.
- Throw away what is in the 3-liter jar
- Transfer the 2 from the 5-liter jar to the 3-liter jar
  - There are 1 left in the 3-liter jar (-1 to 3).
- Refill the 5-liter jar
- Fill the 3-liter jar to the top, what is left in the 5-liter jar is 4 liters

http://www.educaplanet.com/educaplanet/2014/11/acertijo-logica-las-jarras-de-agua/

Exercise

2 minutes máx.

- For the numbers 112 and -71 in decimal base,
  do the sum in complement to the base (base 10).

Exercise (solution)

2 minutes máx.

- Base complement of -71 is:

```
 1000
- 071
--------
  929
```

- The sum is:

```
  112
  929
--------
 ~~1~~041
```

```
  112
 -071
--------
  041
```


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1. Introduction
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      2. **Strings**
   2. Numerical
      1. Natural and integer
      2. Fixed point
      3. Floating point (IEEE 754 standard)

Alphanumeric representation

- Each character is encoded as one byte.
- With n bits ⇒ up to 2^n characters can be encoded:

# bits	# characters	Includes…	Example
6	64	• 26 letter: a...z • 10 number: 0...9 • punctuation: . , ; : ... • specials: + - [...	***BCDIC***
7	128	• adds uppercases and control characters	***ASCII***
8	256	• adds accented letters, ñ, semigraphic characters	***EBCDIC*** ***ASCII extended***
16	34.168	• add support for Chinese, Arabic, ...	***UNICODE***

Example: ASCII table (7 bits)

ASCII value	Character	Control character	ASCII value	Character	ASCII value	Character	ASCII value	Character
000	(null)	NUL	032	(space)	064	@	096	
001	☺	SOH	033	!	065	A	097	a
002	☻	STX	034	"	066	B	098	b
003	♥	ETX	035	#	067	C	099	c
004	♦	EOT	036	$	068	D	100	d
005	♣	ENQ	037	%	069	E	101	e
006	♠	ACK	038	&	070	F	102	f
007	(beep)	BEL	039	'	071	G	103	g
008	◘	BS	040	(	072	H	104	h
009	(tab)	HT	041	)	073	I	105	i
010	(line feed)	LF	042	*	074	J	106	j
011	(home)	VT	043	+	075	K	107	k
012	(form feed)	FF	044	,	076	L	108	l
013	(carriage return)	CR	045	-	077	M	109	m
014	♫	SO	046	.	078	N	110	n
015	☼	SI	047	/	079	O	111	o
016	►	DLE	048	0	080	P	112	p
017	◄	DC1	049	1	081	Q	113	q
018	↕	DC2	050	2	082	R	114	r
019	‼	DC3	051	3	083	S	115	s
020	π	DC4	052	4	084	T	116	t
021	§	NAK	053	5	085	U	117	u
022	▬	SYN	054	6	086	V	118	v
023	↨	ETB	055	7	087	W	119	w
024	↑	CAN	056	8	088	X	120	x
025	↓	EM	057	9	089	Y	121	y
026	→	SUB	058	:	090	Z	122	z
027	←	ESC	059	;	091	[	123	{
028	(cursor right)	FS	060	<	092	\	124	¦
029	(cursor left)	GS	061	=	093	]	125	}
030	(cursor up)	RS	062	>	094	^	126	~
031	(cursor down)	US	063	?	095	_	127	⌂

Example: ASCII table (7 bits)

control characters

ASCII value	Character	Control character	ASCII value	Character	ASCII value	Character	ASCII value	Character
000	(null)	NUL	032	(space)	064	@	096	
001	☺	SOH	033	!	065	A	097	a
002	☻	STX	034	"	066	B	098	b
003	♥	ETX	035	#	067	C	099	c
004	♦	EOT	036	$	068	D	100	d
005	♣	ENQ	037	%	069	E	101	e
006	♠	ACK	038	&	070	F	102	f
007	(beep)	BEL	039	'	071	G	103	g
008	◘	BS	040	(	072	H	104	h
009	(tab)	HT	041	)	073	I	105	i
010	(line feed)	LF	042	*	074	J	106	j
011	(home)	VT	043	+	075	K	107	k
012	(form feed)	FF	044	,	076	L	108	l
013	(carriage return)	CR	045	-	077	M	109	m
014	♫	SO	046	.	078	N	110	n
015	☼	SI	047	/	079	O	111	o
016	►	DLE	048	0	080	P	112	p
017	◄	DC1	049	1	081	Q	113	q
018	↕	DC2	050	2	082	R	114	r
019	‼	DC3	051	3	083	S	115	s
020	π	DC4	052	4	084	T	116	t
021	§	NAK	053	5	085	U	117	u
022	▬	SYN	054	6	086	V	118	v
023	↨	ETB	055	7	087	W	119	w
024	↑	CAN	056	8	088	X	120	x
025	↓	EM	057	9	089	Y	121	y
026	→	SUB	058	:	090	Z	122	z
027	←	ESC	059	;	091	[	123	{
028	(cursor right)	FS	060	<	092	\	124	¦
029	(cursor left)	GS	061	=	093	]	125	}
030	(cursor up)	RS	062	>	094	^	126	~
031	(cursor down)	US	063	?	095	_	127	⌂

< 32

Example: ASCII table (7 bits)

distance between uppercase and lowercase letters

ASCII value	Character	Control character	ASCII value	Character	ASCII value	Character	ASCII value	Character
000	(null)	NUL	032	(space)	064	@	096	
001	☺	SOH	033	!	065	A	097	a
002	☻	STX	034	"	066	B	098	b
003	♥	ETX	035	#	067	C	099	c
004	♦	EOT	036	$	068	D	100	d
005	♣	ENQ	037	%	069	E	101	e
006	♠	ACK	038	&	070	F	102	f
007	(beep)	BEL	039	'	071	G	103	g
008	◘	BS	040	(	072	H	104	h
009	(tab)	HT	041	)	073	I	105	i
010	(line feed)	LF	042	*	074	J	106	j
011	(home)	VT	043	+	075	K	107	k
012	(form feed)	FF	044	,	076	L	108	l
013	(carriage return)	CR	045	-	077	M	109	m
014	♫	SO	046	.	078	N	110	n
015	☼	SI	047	/	079	O	111	o
016	►	DLE	048	0	080	P	112	p
017	◄	DC1	049	1	081	Q	113	q
018	↕	DC2	050	2	082	R	114	r
019	!!	DC3	051	3	083	S	115	s
020	π	DC4	052	4	084	T	116	t
021	§	NAK	053	5	085	U	117	u
022	▬	SYN	054	6	086	V	118	v
023	↨	ETB	055	7	087	W	119	w
024	↑	CAN	056	8	088	X	120	x
025	↓	EM	057	9	089	Y	121	y
026	→	SUB	058	:	090	Z	122	z
027	←	ESC	059	;	091	[	123	{
028	(cursor right)	FS	060	<	092	\	124	¦
029	(cursor left)	GS	061	=	093	]	125	}
030	(cursor up)	RS	062	>	094	^	126	~
031	(cursor down)	US	063	?	095	_	127	⌂

97-65=32

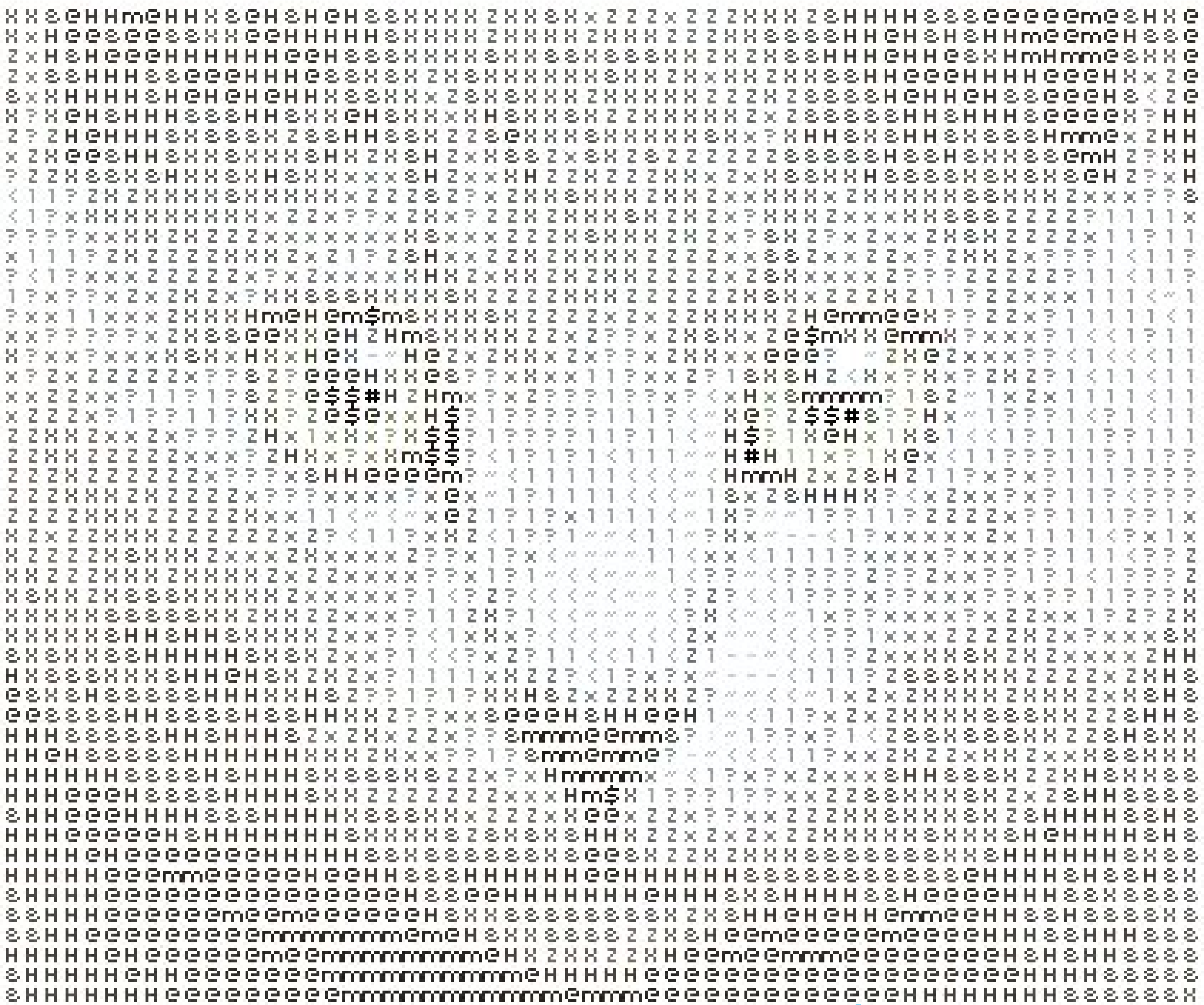
Example: ASCII table (7 bits)

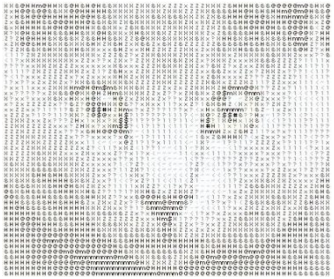
conversion of a number to a character

ASCII value	Character	Control character	ASCII value	Character	ASCII value	Character	ASCII value	Character
000	(null)	NUL	032	(space)	064	@	096	
001	☺	SOH	033	!	065	A	097	a
002	☻	STX	034	"	066	B	098	b
003	♥	ETX	035	#	067	C	099	c
004	♦	EOT	036	$	068	D	100	d
005	♣	ENQ	037	%	069	E	101	e
006	♠	ACK	038	&	070	F	102	f
007	(beep)	BEL	039	'	071	G	103	g
008	◘	BS	040	(	072	H	104	h
009	(tab)	HT	041	)	073	I	105	i
010	(line feed)	LF	042	*	074	J	106	j
011	(home)	VT	043	+	075	K	107	k
012	(form feed)	FF	044	,	076	L	108	l
013	(carriage return)	CR	045	-	077	M	109	m
014	♫	SO	046	.	078	N	110	n
015	☼	SI	047	/	079	O	111	o
016	►	DLE	048	0	080	P	112	p
017	◄	DC1	049	1	081	Q	113	q
018	↕	DC2	050	2	082	R	114	r
019	‼	DC3	051	3	083	S	115	s
020	π	DC4	052	4	084	T	116	t
021	§	NAK	053	5	085	U	117	u
022	▬	SYN	054	6	086	V	118	v
023	↨	ETB	055	7	087	W	119	w
024	↑	CAN	056	8	088	X	120	x
025	↓	EM	057	9	089	Y	121	y
026	→	SUB	058	:	090	Z	122	z
027	←	ESC	059	;	091	[	123	{
028	(cursor right)	FS	060	<	092	\	124	¦
029	(cursor left)	GS	061	=	093	]	125	}
030	(cursor up)	RS	062	>	094	^	126	~
031	(cursor down)	US	063	?	095	_	127	⌂

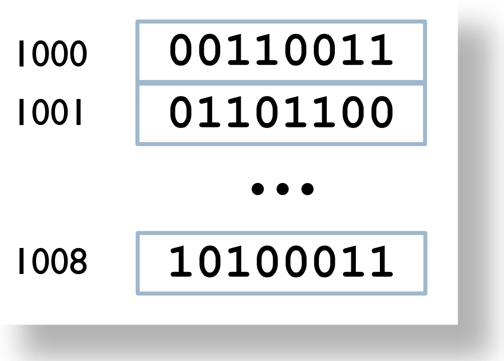
6+48=54

Curiosity:
Display “image” with characters



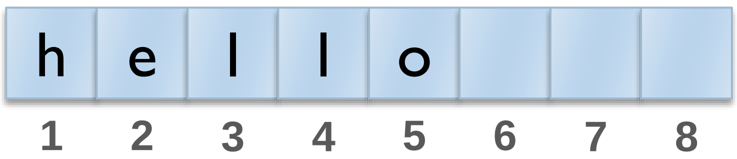


http://www.typorganism.com/asciiomatic/

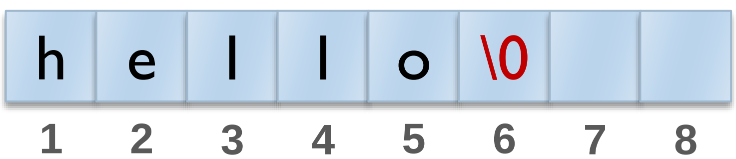


Character strings

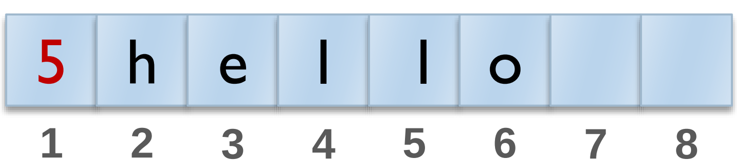
1. Fixed-length string:



2. Variable-length string with delimiter:



3. Variable-length strings with length in header:



Contents

Numerical representation

- Classification of real numbers:
  - Naturals: 0, 1, 2, 3, ...
  - Integers: ... -3, -2, -1, 0, 1, 2, 3, ...
  - Rational: fractions (5/2 = 2,5)
  - Irrational: $2^{1/2}$, π, e, ...

- Infinite sets but finite representation space:
  - Impossible to represent all ☹

- Characteristics of the representation used:
  - Represented element:
    Natural, integer, …
  - Representation range:
    Interval between minor and major not representable
  - Resolution of representation:
    Difference between a representable number and the following one.
    It represents the maximum error committed. It can be cte. or variable.

Most used binary representation systems

A. (Pure) binary — natural

B. Sign-Magnitude

C. One's complement (Ca 1)

D. Two's complement (Ca 2) — integer

E. Biased $2^{n-1}-1$

F. Floating point: IEEE 754 standard — real

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(Pure) binary or unsigned binary [natural numbers]

- Positioning system with base 2 and without fractional part.

n-1 0

n bits

$$V(X) = \sum_{i=0}^{n-1} 2^{i} \cdot x_{i}$$

- Representation range: $[0, 2^{n} - 1]$
- Resolution: 1 unit

Comparative example (3 bits)

Decimal	Pure Binary
+7	111
+6	110
+5	101
+4	100
+3	011
+2	010
+1	001
+0	000
-0	N.D.
-1	N.D.
-2	N.D.
-3	N.D.
-4	N.D.
-5	N.D.
-6	N.D.
-7	N.D.

Signed binary number or Sign-Magnitude [integer numbers]

- One bit (S) is reserved for the sign (0 ⇒ +; 1 ⇒ -)

n-1	n-2 ... 0
S	Magnitude (n-1 bits)

$$\text{Si } x_{n-1} = 0 \quad V(X) = \sum_{i=0}^{n-2} 2^i \cdot x_i$$

$$\text{Si } x_{n-1} = 1 \quad V(X) = -\sum_{i=0}^{n-2} 2^i \cdot x_i$$

$$\Rightarrow V(X) = (1 - 2 \cdot x_{n-1}) \cdot \sum_{i=0}^{n-2} 2^i \cdot x_i$$

- Representation range: $[-2^{n-1}+1, \; 2^{n-1}-1]$
- Resolution: 1 unit
- Ambiguity of zero + complex hw. for subtraction

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Comparative example (3 bits)

Decimal	Pure Binary	Sign-Magnitude
+7	111	N.D.
+6	110	N.D.
+5	101	N.D.
+4	100	N.D.
+3	011	011
+2	010	010
+1	001	001
+0	000	000
-0	N.D.	100
-1	N.D.	101
-2	N.D.	110
-3	N.D.	111
-4	N.D.	N.D.
-5	N.D.	N.D.
-6	N.D.	N.D.
-7	N.D.	N.D.

Example

- Can we represent 745_{10} in sign-magnitude with 10 bits?

Example (solution)

- Can we represent 745_{10} in sign-magnitude with 10 bits?

- With 10 bits the range in sign-magnitude is:
  $[-2^9+1,\ldots,-0,+0,\ldots.2^9-1] \Rightarrow [-511, 511]$
  then, **we cannot represent** 745

One's complement (to the base minus one) [integer] (1/3)

- **Positive number**:
  is represented in pure binary with n-1 bits

n-1	n-2 ... 0
0	Magnitude

$$V(X) = \sum_{i=0}^{n-1} 2^i \cdot x_i = \sum_{i=0}^{n-2} 2^i \cdot x_i$$

- Representation range (+): $[0, 2^{n-1} - 1]$
- Resolution: 1 unit

One's complement (to the base minus one) [integer] (2/3)

- **Negative number**:
  - Complemented to the base minus one.
  - The number X < 0 is represented as $2^n - X - 1$ with n bits

n-1	n-2 … 0
1	1'C of magnitude

$$V(X) = -2^n + \sum_{i=0}^{n-1} 2^i \cdot y_i + 1$$

- Representation range (-): $[-(2^{n-1}-1), -0]$
- Resolution: 1 unit

One's complement (to the base minus one) [integer] (3/3)

Tip: C a 1 (X) = X
C a 1 (-X) = change the 1's to 0's and the 0's to 1's

- Example: For n=4 ⇒ the value $+3_{10} = 0011_2$
- Example: For n=4 ⇒ the value $-3_{10} = 1100_2$
  - \- ⇒ 1 (sign bit and also part of magnitude)
  - C a 1(3) ⇒ $2^4 - 0011_2 - 1 = 2^4 - 3 - 1 = 12$ ⇒ 1100_2

- Representation range: $[-2^{n-1}+1, 2^{n-1}-1]$
- Resolution: 1 unit
- Zero has a double representation (+0 y -0)
- Symmetrical range

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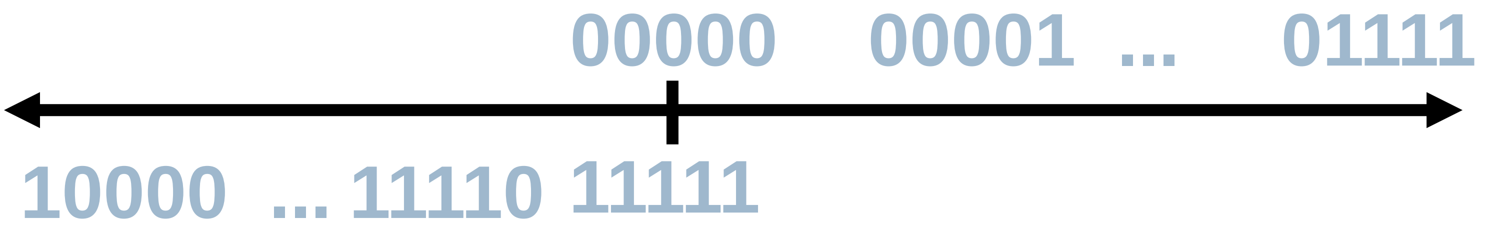
Ones' complement

- Positive numbers have a 0 in the most significant bit.



- Negative numbers have a 1 in the most significant bit.

Comparative example (3 bits)

Decimal	Pure Binary	Sign-Magnitude	One's complement
+7	111	N.D.	N.D.
+6	110	N.D.	N.D.
+5	101	N.D.	N.D.
+4	100	N.D.	N.D.
+3	011	011	011
+2	010	010	010
+1	001	001	001
+0	000	000	000
-0	N.D.	100	111
-1	N.D.	101	110
-2	N.D.	110	101
-3	N.D.	111	100
-4	N.D.	N.D.	N.D.
-5	N.D.	N.D.	N.D.
-6	N.D.	N.D.	N.D.
-7	N.D.	N.D.	N.D.

Example

With n = 5 bits and using one's complement:

- How is represented X = 5?

- How is represented X = -5?

- What is the value of 00111 in 1's complement?

- What is the value of 11000 in 1's complement?

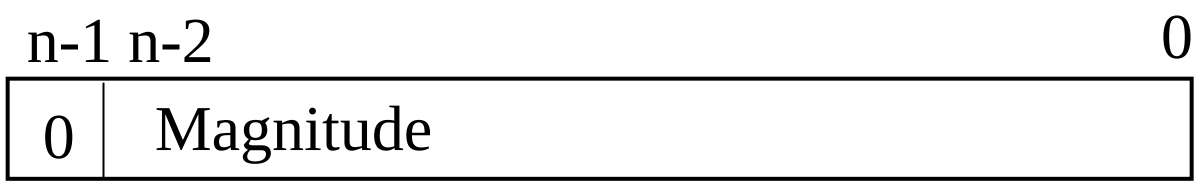
Example (solution)

With n = 5 bits and using one's complement:

- How is represented X = 5?
  - Because is positive then is like (pure) binary
    - 00101
- How is represented X = -5?
  - Because is negative, then 5 is complemented to one (00101)
    - 11010
- What is the value of 00111 in 1's complement?
  - Because is positive then its value is 7
- What is the value of 11000 in 1's complement?
  - Because is negative, then is complemented and is 00111 (7)
    - The value is -7

Two's complement (complement to the base) [integer] (1/3)

- **Positive number**:
  is represented in pure binary with n-1 bits



$$V(X) = \sum_{i=0}^{n-1} 2^i \cdot x_i = \sum_{i=0}^{n-2} 2^i \cdot x_i$$

- Representation range (+): $[0, 2^{n-1} - 1]$
- Resolution: 1 unit

Two's complement (complement to the base) [integer] (2/3)

- **Negative number**:
  - Complemented to the base.
  - The number $X < 0$ is represented as $2^n - X$ with n bits

n-1	n-2 … 0
1	2'C of the magnitude

$$V(X) = -2^n + \sum_{i=0}^{n-1} 2^i \cdot y_i$$

- Representation range (-): $[-2^{n-1}, -1]$
- Resolution: 1 unit

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Two's complement (complement to the base) [integer] (3/3)

Tip: C a 2 (X) = X
C a 2 (-X) = C a 1 (X) + 1

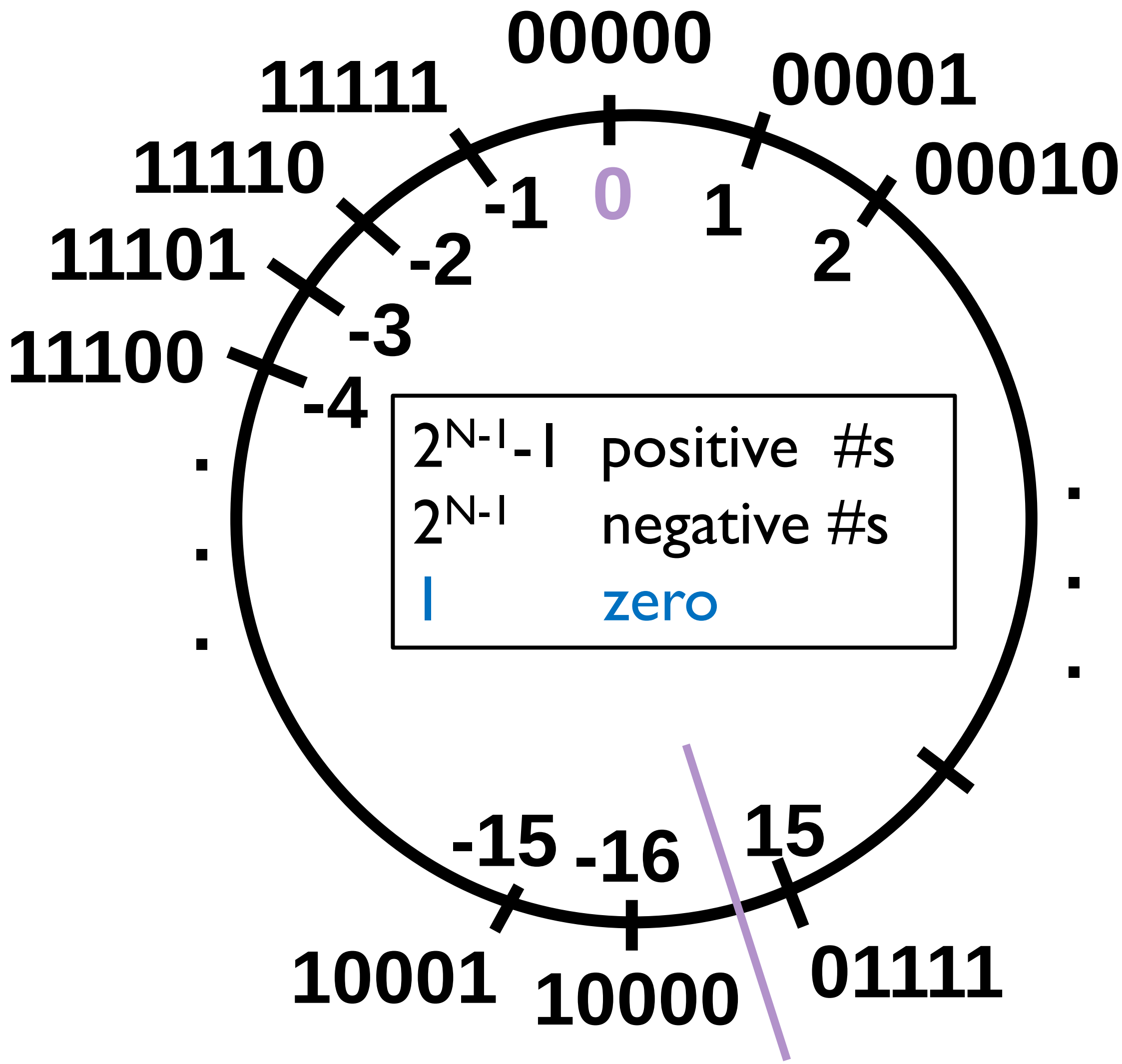
- Example: For n=4 ⇒ + 3 = 0011_2
- Example: For n=4 ⇒ -3 = 1101_2
  - 1 ⇒ - (sign bit and also part of magnitude)
  - C a 2 (3) = C a 2(0011_2) = 2^4 - 3 = 13 ⇒ 1101_2

- Representation range: $[-2^{n-1}, 2^{n-1}-1]$
- Resolution: 1 unit
- 0 has only one representation (∄ -0)
- Asymmetric range

Comparative example (3 bits)

Decimal	Pure Binary	Sign-Magnitude	One's complement	Two's complement
+7	111	N.D.	N.D.	N.D.
+6	110	N.D.	N.D.	N.D.
+5	101	N.D.	N.D.	N.D.
+4	100	N.D.	N.D.	N.D.
+3	011	011	011	011
+2	010	010	010	010
+1	001	001	001	001
+0	000	000	000	000
-0	N.D.	100	111	N.D.
-1	N.D.	101	110	111
-2	N.D.	110	101	110
-3	N.D.	111	100	101
-4	N.D.	N.D.	N.D.	100
-5	N.D.	N.D.	N.D.	N.D.
-6	N.D.	N.D.	N.D.	N.D.
-7	N.D.	N.D.	N.D.	N.D.

Two's complement



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Two's complement with 32-bits

$0000 \ldots 0000\ 0000\ 0000\ 0000_{2c} = 0_{(10}$
$0000 \ldots 0000\ 0000\ 0000\ 0001_{2c} = 1_{(10}$
$0000 \ldots 0000\ 0000\ 0000\ 0010_{2c} = 2_{(10}$
. . .
$0111 \ldots 1111\ 1111\ 1111\ 1101_{2c} = 2{,}147{,}483{,}645_{(10}$
$0111 \ldots 1111\ 1111\ 1111\ 1110_{2c} = 2{,}147{,}483{,}646_{(10}$
$0111 \ldots 1111\ 1111\ 1111\ 1111_{2c} = 2{,}147{,}483{,}647_{(10}$

$1000 \ldots 0000\ 0000\ 0000\ 0000_{2c} = -2{,}147{,}483{,}648_{(10}$
$1000 \ldots 0000\ 0000\ 0000\ 0001_{2c} = -2{,}147{,}483{,}647_{(10}$
$1000 \ldots 0000\ 0000\ 0000\ 0010_{2c} = -2{,}147{,}483{,}646_{(10}$
. . .
$1111 \ldots 1111\ 1111\ 1111\ 1101_{2c} = -3_{(10}$
$1111 \ldots 1111\ 1111\ 1111\ 1110_{2c} = -2_{(10}$
$1111 \ldots 1111\ 1111\ 1111\ 1111_{2c} = -1_{(10}$

Biased $2^{n-1}-1$ representation [integer]

- El valor X con n bits se reprsenta como $X + 2^{n-1}-1$
- Bias refers to the value $2^{n-1}-1$

n-1 0

n bits

$$V(X) = \sum_{i=0}^{n-1} 2^{i} \cdot x_{i} - (2^{n-1} - 1)$$

- Representation range: $[-(2^{n-1} - 1), 2^{n} - 1]$
- Resolution: 1 unit
- No existe ambigüedad con el 0

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Comparative example (3 bits)

Decimal	Pure Binary	Sign-Magnitude	One's complement	Two's complement	Biased-3
+7	111	N.D.	N.D.	N.D.	N.D.
+6	110	N.D.	N.D.	N.D.	N.D.
+5	101	N.D.	N.D.	N.D.	N.D.
+4	100	N.D.	N.D.	N.D.	111
+3	011	011	011	011	110
+2	010	010	010	010	101
+1	001	001	001	001	100
+0	000	000	000	000	011
-0	N.D.	100	111	N.D.	N.D.
-1	N.D.	101	110	111	010
-2	N.D.	110	101	110	001
-3	N.D.	111	100	101	000
-4	N.D.	N.D.	N.D.	100	N.D.
-5	N.D.	N.D.	N.D.	N.D.	N.D.
-6	N.D.	N.D.	N.D.	N.D.	N.D.
-7	N.D.	N.D.	N.D.	N.D.	N.D.

Contents

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Comparison of arithmetic in B, 1C and 2C

	Binary	One's complement	Two' complement
Add	10110 01100 ---------- 100010	same as binary	same as binary
Subtract	10110 01100 ---------- 01010	add and if there is Cn-1 then add Cn-1 to total	add and if there is Cn-1 then discard it

In hardware, it is easier to operate with complement

Comparison of arithmetic in B, 1C and 2C

why add the carry to the result in 1C

- -X is represented as $2^n - X - 1$
- -Y is represented as $2^n - Y - 1$
- -(X + Y) is represented as $2^n - (X+Y) - 1$

- -(X + Y) the operation gives $2^n + 2^n - (X + Y) - 2$ + 1

	Bin…		…ment
Add			
Subtract	10110 01100 ---------- 01010	add and if there is Cn-1 then add Cn-1 to total	add and if there is Cn-1 then discard it

Correction of the result by adding the carry...

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Comparison of arithmetic in B, 1C and 2C

why discard the carry in 2C

	Bi…		…ent
Add			
Subtract	10110 01100 ---------- 01010	add and if there is Cn-1 then add Cn-1 to total	add and if there is Cn-1 then discard it

- -X is represented as $2^n - X$
- -Y is represented as $2^n - Y$
- -(X + Y) is represented as $2^n - (X+Y)$
- -(X + Y) the operation gives $2^n + 2^n - (X + Y)$

Correction of the result by discarding the carry…

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Comparison of arithmetic in B, 1C and 2C

	Binary	One's complement	Two' complement
Detect overflow	The result needs 1 bit more There are Cn	Adding ++ is –, Adding – – is + Cn <> Cn-1	Adding ++ is –, Adding – – is + Cn <> Cn-1
Sign extension	0…0 10110	1…1←10110 0…0←00110	1…1←10110 0…0←00110
…	…	…	…

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ARCOS Group

Universidad Carlos III de Madrid

Lesson 2 (I)
Representation of information

Computer Structure

Bachelor in Computer Science and Engineering

Example

Indicate the representation of the following numbers, giving a brief justification of your answer:

1. **-32** in one's complement with **6 bits**
2. **-32** in two's complement with **6 bits**
3. **-10** in sign-magnitude with **5 bits**
4. **+14** in two's complement with **5 bits**

Example (solution)

1. With 6 bits **is not representable** in 1C: $[-2^{6-1}+1,\ldots,-0,+0,\ldots 2^{6-1}-1]$

2. 1C + 1 -> **100000**

3. Sign=1, magnitude=1010 -> **11010**

4. Positive -> 1C=2C=SM -> **01110**

Example
Arithmetic in 1's complement

- With n = 5 bits
- X = 5
  - In one's complement = 00101
- Y = 7
  - In one's complement = 00111
- ¿X + Y?

X	=	00101
Y	=	00111+
X+Y	=	01100

- The value 01100 in one's complement is 12

Example
Arithmetic in 1's complement

- With n = 5 bits
- X = -5
  - In one's complement = complement of 00101: 11010
- Y = -7
  - In one's complement = complement of 00111: 11000
- ¿X + Y?

```
-X       =   11010
-Y       =   11000+
-(X+Y)   =  110010        A carry is generated and is added
                  1
              10011
```

- The value of 10011 in one's complement is negative and the complement is -01100 = - 12

Example
Arithmetic in 2's complement

- With n = 5 bits
- X = 5
  - In two's complement = 00101
- Y = 7
  - Is two's complement = 00111
- X + Y?

X	=	00101
Y	=	00111+
X+Y	=	01100

- The value of 01100 in two's complement is 12

Example
Arithmetic in 2's complement

- With n = 5 bits
- X = -5
  - In two's complement = 11010 + 1 = 11011
- Y = -7
  - In two's complement = 11000 +1 = 11001
- X + Y?

```
-X       =   11011
-Y       =   11001+
-(X+Y)   =  110100       discard the carry
```

- The result is 10100. The value is 01011 + 1 = 01100 = >- 12

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Ejemplo
Aritmética en complemento a dos

- With n = 5 bits
- X = 8
  - In two's complement = 01000
- Y = 9
  - In two's complement = 01001
- ¿X + Y?

X	=	01000
Y	=	01001+
X+Y	=	10001

- A negative value is obtained ⇒ overflow

Ejemplo
Aritmética en complemento a dos

- With n = 5 bits
- X = -8
  - In two's complement = 10111 + 1 = 11000
- Y = -9
  - In two's complement = 10110 +1 = 10111
- X + Y?

-X	=	11000	
-Y	=	10111+	
-(X+Y)	=	101111	The carry is discarded

- The result 01111, is positive $\Rightarrow$ overflow

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Extensión de signo en complemento a dos

- How to represent the same number of *n* bits but with *m* bits, being $n < m$?

- Example:
  - n = 4, m = 8
  - X = 0110 with 4 bits ⇒ X = 00000110 with 8 bits
  - X = 1011 with 4 bits ⇒ X = 11111011 with 8 bits

Example

- Using 5 bits, compute the followingg additions in 1's complement:
  a) 4 +12
  b) 4 -12
  c) -4 -12

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Example (solution)

- By using 5 bits in 1's complement the result is:

a) 4 +12

00100
01100

10000 $\Rightarrow$ -15 $\Rightarrow$ negative! $\Rightarrow$ overflow

Example (solution)

- By using 5 bits in 1's complement the result is:
  - b) 4 - 12

```
00100
10011
--------
10111 ⇒ -8
```

Example (solution)

- By using 5 bits in 1's complement the result is:
  c) -4 - 12

```
      11011
      10011
      --------
     101110 ⇒ 6 bits are needed ⇒ overflow
```